

MTH 221: Linear Algebra, Exam One

Ayman Badawi

QUESTION 1. a) Given $A = \begin{bmatrix} 3 & a & -2 \\ k & b & c \\ 4 & d & 4 \end{bmatrix}$ such that $det(A) = -30$ and $F = \begin{bmatrix} 3 & a & -2 \\ k & b+4 & c \\ 4 & d & 4 \end{bmatrix}$. Find $det(F)$.

b) Given $C = \begin{bmatrix} 1 & 2 & 4 & 6 \\ -1 & b_1 & b_2 & b_3 \\ 2 & 4 & c_1 & c_2 \\ 3 & 6 & 12 & d_1 \end{bmatrix}$ such that $C \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = C_3$ has a unique solution (recall that C_3 is the third column of C).

i) What is the solution-set of the system? (i.e., find the values of x_1, x_2, x_3, x_4)

ii) Find all possible values for $b_1, b_2, b_3, c_1, c_2, d_1$?

QUESTION 2. a)Given that A and B are NONZERO $n \times n$ matrices for some $n > 1$ such that AB is a zero matrix. Convince me CLEARLY that A AND B are non-invertible (singular).

b) Given A ia a 9×9 matrix such that $A^2 = I_9$ but $A \neq I_9$ and $A \neq -I_9$. Convince me CLEARLY that $A + I_9$ AND $A - I_9$ are non-invertible.

c) Consider the system :

$$x_1 + x_2 + x_3 - 3x_4 = 2$$
$$-2x_1 - 2x_2 + kx_3 + 6x_4 = 10$$
$$-x_1 - x_2 - x_3 + 3x_4 = k$$

.

Is it possible that the system be consistent for some values of k? If yes or no, convince me CLEARLY that your conclusion is correct.

QUESTION 3. Find a 2×3 matrix A such that

$$\left(\begin{bmatrix} 0 & -3 \\ -3 & 2 \end{bmatrix} A\right)^T + 2A^T = \begin{bmatrix} 0 & 2 \\ 1 & 0 \\ 0 & 4 \end{bmatrix}$$

QUESTION 4. Find the solution-set for the following system

$$x_1 + x_3 - x_4 = 2$$

$$-2x_1 + x_2 - x_3 + 2x_4 = 0$$

$$-2x_2 - x_3 + x_4 = 4$$

QUESTION 5. Let $A = \begin{bmatrix} 4 & -2 & 6 \\ -2 & 3 & 6 \\ -8 & 4 & 7 \end{bmatrix}$. Use Cramer to find the value of x_3 in the system $AX = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix}$

QUESTION 6. Given A is 4×4 and

$$A \quad \overline{2R_1} \quad B \quad \overline{-2R_1 + R_3 \to R_3} \quad C \quad \overline{0.25R_3} \quad D = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

a) Find $det(A)$

b) Find $det(2AD^T)$

c) Find $det(-A^{-1}B)$

Question 6 CONTINUES

d) Find Two Elementary MATRICES, E, F such that $EFD = B$

e) Find A^{-1} [Hint : You do not need to find A, STARE Clearly !!!, however you may ignore this hint and do whatever]

f) Solve the system $A^T X = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$

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Exam II , MTH 221 , Fall 2012

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TOTAL POINTS = 90

QUESTION 1. (10 points) Given A ia an invertible 2×2 matrix such that $A^{-1} = 0.25A + 0.75I_2$ and $A \neq cI_2$ for some constant c.

(i) Find the eigenvalues of A. [Hint: Assume if $f(\alpha)$ is a polynomial of degree 2 such that the coefficient of α^2 is 1 and $f(A)$ = zero matrix, then $f(\alpha) = C_A(\alpha)$]

(ii) Find $det(A)$.

QUESTION 2. (12 points) Let A be an invertible 2×2 matrix such that $-2, 2$ are the eigen-values of A such that $E_2 = \{(2x_1, x_1) \mid x_1 \in R\}$ and $E_{-2} = \{(-2x_1, 0) \mid x_1 \in R\}$.

(i) Find a diagonal matrix D and an invertible matrix M such that $MDM^{-1} = A^{-1}$.

(ii) Find an invertible matrix N and a diagonal matrix F such that $NFN^{-1} = A^T$

(iii) Let $B = A + 3I_2$. Find $N(B)$.

QUESTION 3. (12 points) Give A is a 3×3 matrix such that $C_A(\alpha) = (\alpha + 1)^2(\alpha - 2)$, $E_{-1} = \{(-2x_2, -x_2, 0) \mid x_2 \in R\}$ and $E_2 = \{(2x_3, -x_3, x_3) \mid x_3 \in R\}$

(i) is A diagnolizable? if yes find an invertible matrix Q and a diagonal matrix D such that $QDQ^{-1} = A$. If no, then explain

(ii) Find a nonzero 3×4 matrix F, such that $AF = -F$

(iii) Let $B = A - I_3$. Find a basis for $N(B)$.

QUESTION 4. (10 points) Let $A = \begin{bmatrix} 4 & b & 8 \\ a & 6 & 2 \\ 6 & c & 0 \end{bmatrix}$. Given A is row-equivalent to $\begin{bmatrix} 0 & 0 & 4 \\ 2 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

(i) Find the values of b, a, c.

(ii) Find a basis for the column space of A.

QUESTION 5. (8 points) Let $A = \{(b+2c, 2b+4c, -b-2c) \mid b, c \in R\}$. Is A a subspace of R^3? explain. If yes, find dim(A), basis for A and write A as a span of a basis.

QUESTION 6. (12 points) a) Let $A = \{F \in R^{2\times 2} \mid Rank(F) \leq 1\}$. Then A is not a subspace of $R^{2\times 2}$. Why?

b) Let $A = \{(a, b, c) \mid a + 2b + c + 1 = 0\}$. Then A is not a subspace of R^3. Why?

c) Let $A = \{f(x) \in P_4 \mid f(1) = 0 \text{ or } f(-1) = 0\}$. Then A is not a subspace of P_4. Why?

QUESTION 7. (8 points) Let $A = \{\begin{bmatrix} 2a + -b & -2a \\ 2b & 4a + b \end{bmatrix} \mid a, b \in R\}$. Then A is a subspace of $R^{2\times 2}$. Why? Find dim(A). Find a basis for A. Is $K = \begin{bmatrix} 13 & -12 \\ -2 & 23 \end{bmatrix}$ belongs to A? Explain

QUESTION 8. (18 points) Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 2 & 2 & 3 \end{bmatrix}$

a) Find $N(A)$ and write $N(A)$ as a span of a basis for $N(A)$

b) Find a basis for $Row(A)$ and a basis for $Col(A)$.

c) Let $B \in R^3$. Is the system $AX = B^T$ consistent? if yes, does it have a unique solution or infinitely many solutions? Explain. Is it possible that for every $B \in R^3$, the system $AX = B^T$ has a solution where $x_2 = x_4 = 0$? explain

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Final Exam , MTH 221 , Fall 2012

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[**Circle one** @12 **or** @2, **MAKE SURE YOU HAVE 9 PAGES,** **SCORE** $\overline{100}$].

(i) **(12 points)** Let

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}.$$

a. Find A^{-1}, i.e., the inverse of the matrix A.

b. Use part (a) to solve for M

$$(A^{-1})^T M = \begin{bmatrix} -1 & -3 \\ 5 & 0 \\ 3 & -2 \end{bmatrix}.$$

(ii) **(9 points)** Let

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 5 & 3 & 0 & 0 \\ 3 & -20 & 1 & 0 \\ 3 & -21 & -6 & -2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -2 & 8 & 2 & 4 \\ 0 & 1 & 6 & 3 \\ 0 & -1 & -7 & -2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

a. Compute $det(AB)$.

b. Compute $det(A + B)$.

(iii) **(11 points)** Let $v_1 = (3, -2, 4)$, $v_2 = (-6, 4, -8)$, $v_3 = (-3, 2, 4)$, and let $W = span\{v_1, v_2, v_3\}$.

a. Find a basis for W and determine its dimension.

b. Determine whether $(-12, 8, 0)$ is a linear combination of v_1, v_2 and v_3.

(iv) **(15 points)** Determine if each of the following are linearly independent:

a. $\left\{ \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}, \begin{bmatrix} -3 & 0 \\ -6 & 9 \end{bmatrix} \right\}$, in $R^{2\times 2}$

b. $\{2x^2 + x - 1, x + 1, x^2 + x\}$, in P_3.

c. the columns of $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 1 \\ 5 & 4 & 4 \end{bmatrix}$, in R^3

(v) **(11 points)**

a. Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be the linear transform defined by

$$T(x_1, x_2, x_3, x_4) = (x_1 + x_2 + 2x_3 - 2x_4\ ,\ 2x_3 + x_4)\,.$$

Find the standard matrix representation of T.

b. Find a basis for $Ker(T)$ (Kernel of T) and a basis for $Range(T)$ (the range of T).

(vi) **(12 points)** Let

$$A = \begin{bmatrix} 0 & -1 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

a. Find $C_A(\alpha)$, the characteristic polynomial of A.

b. Find the eigenvalues and the corresponding eigenspaces of A (i.e., for each eigenvalue α, find E_α).

c. Find an invertible matrix Q and a diagonal matrix D such that $QDQ^{-1} = A$. [you do not have to calculate Q^{-1}].

(vii) **(10 points)** Suppose that A is an invertible $n \times n$ matrix and $\mathbf{v} \in R^n$ is an eigenvector for A with corresponding eigenvalue 6. Show that $\mathbf{v}$ is also an eigenvector for $B = A^2 + 12A^{-1}$. What is the corresponding eigenvalue in this case?

(viii) **(8 points)**

a. Let $u = (3,\ 0,\ 4) \in R^3$ Find $\|u\|$. The length (norm) of u.

b. Find a scalar c, so that $v = (c, 10,\ 6)$ is orthogonal to u.

(ix) ((**12 points**) Let $D = span\{\ (1,0,1,1), (-1,0,1,1), (1,0,-1,-2)\}$.

Use the Gram-Schmidt method to find an orthogonal basis for D.

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